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A methodological framework for designing dynamic heat price for demand response in district heating

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ABSTRACT

Currently, the price per consumption of most district heating consumers is static, providing no incentive for consumers to change their heating behavior. A dynamic pricing structure can encourage consumers to adjust their heat demand according to supply conditions. This paper presents a methodology for designing day-ahead dynamic price profiles for district heating to encourage buildings for certain demand response purposes effectively. This method relies on a method to characterize the energy flexibility of buildings and an inverse optimization to obtain the optimal dynamic price profile. The methodology is tested on a residential neighborhood as a case study, using a virtual experiment that includes buildings and a district heating network. Results show that the designed dynamic price could encourage consumers to change their demand profile to match a pre-defined load-shifting profile with 16.6 % MAPE. Accordingly, the peak load was reduced by 84.4 % as a result of responding to a tailored price profile, considering the flexibility potential of the neighborhood. In addition, heat costs in the neighborhood were reduced by 46.6 % compared to the flat price. The findings of this paper highlight the benefits of the dynamic heat price for district heating operators and consumers over the flat price.

1. Introduction

According to the International Energy Agency, heating and cooling account for 50 % of the world's energy consumption, contributing to 40 % of global carbon dioxide emissions [1]. Natural gas, coal, wood, etc., are burnt daily to meet buildings' heating demands. With population growth, political conflicts, and problems associated with climate change, it is crucial to rely on renewable energy and other clean energy sources [2]. District heating can optimally utilize locally available heat sources and waste heat [3], which can be dissipated heat from industrial processes, incineration of household waste, agricultural residues, and heat from processes in Combined Heat and Power plants (CHP) [4]. District heating is identified to be a flexible resource for the electricity grid, using the thermal storage of the network [5]. According to Danish Energy Agency statistics, district heating was the main heating source of 66 % of residential dwellings in Denmark in 2022 [6]. Decarbonizing the heating sector has been the goal of many cities [7]. For example, Sønderborg has set a goal to decarbonize its heating sector by 2035 [8].

During peak demand periods, clean district heating plants (e.g., waste incineration plants, waste heat from data centers, renewable energy) may be insufficient to cover the demand and backup fossil-based boilers are usually used [9]. There are generally three solutions to prevent using these boilers: 1) increasing penetration of clean energy sources in district heating, 2) adding large thermal storage units to store heat for peak demand periods, and 3) using demand response and load shifting on the consumer side. Among these solutions, demand response is proven to be a successful strategy with lower investment costs [10]. Buildings can store heat for up to many hours due to the large thermal mass. Indoor setpoint control in buildings is recognized as a short-term storage solution, which can benefit district heating systems by solving local network congestion and peak demand challenges by reducing peaks [11]. District heating companies can send price signals to heat consumers to indirectly control indoor setpoints. A challenge is to estimate how buildings would react to different price signals, just as mentioned in Ref. [12]. The first studies on this topic started by characterizing energy flexibility in a static form or pre-specified profiles [13] and introduced the term *Flexibility Function*, which made a linear link between price and change in demand [14]. However, the linearity assumption hindered the application of the flexibility function and it was further developed as a nonlinear flexibility function [15].

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Many studies identified demand response as a successful and promising solution to overcome mismatches between demand and supply in district heating [16]. One effective way of unlocking demand response in district heating is shifting from a flat price to a dynamic price. Dynamic price can indirectly control heat load of a building by incentivizing consumers to change their heating behavior and gain economic benefits from it [17]. Additionally, in Ref. [18], authors comprehensively define concepts of next-generation district heating systems, in which they propose the change from the prevailing flat price to a varying price as one of the critical elements for decarbonization. To the best of the authors' knowledge, there are a limited number of papers dedicated to analyzing dynamic prices in district heating systems for demand response purposes, with very few papers giving methods for designing dynamic heat prices. Li et al. [19] created an Elman Neural Networks (ENN) model for predicting heat demands and calculated the price based on that, focusing on the expense of heat customers under different price models: seasonal price model, subscription price model, and real price models. They concluded that by using the dynamic price schemes, district heating can obtain a lower overall cost compared with current price schemes. Hua et al. [20] implemented an integrated demand response approach consisting of an optimization layer and a control layer for an education building. They calculated the true cost of producing heat by the air source heat pump using electricity prices and other influencing factors and took that as dynamic heat price. Their results show a 9 % reduction in energy consumption and 13 % lower heat costs. Bai et al. [21] formulated the heating market as a Stackelberg game with a bi-level structure, the upper-level problem being the heat provider aiming to find the dynamic heat price to maximize the net profit, and the lower level being consumers to find the optimum heat demand profile to minimize their costs. They applied the approach to a groundwater-based low-temperature district heating system, comprising one heat pump as the heat provider, a thermal storage, and three commercial buildings as consumers. Their study demonstrated that dynamic heat price can be a successful solution to utilize the flexibility of heat demand.

As [22] suggests, new tariffs and business models are required for district heating operators to incentivize consumers for demand response purposes. Shifting from a flat heat price to a dynamic heat price can be a solution for the widespread activation of demand response. Many recognize the advantages of dynamic price for district heating, yet they lack implementation methodologies. Marginal heat production cost has been widely used in the literature as dynamic heat price for control purposes of buildings and neighborhoods. For example, Foteinaki et al. [23] took the marginal heat production cost of a district heating company as a dynamic heat price to trigger demand response actions in a multifamily apartment and evaluate its energy flexibility potential. Cai et al. [24] also used the marginal cost of the district heating network in Copenhagen as a dynamic heat price for optimal energy scheduling of the network. Dominković et al. [25] analyzed waste heat utilization in district heating networks of Nordic countries, using a dynamic heat price based on marginal heat production cost. Although district heating pricing based on marginal cost benefits operators and consumers, it cannot act as a proper demand response solution as it fails to successfully motivate consumers for load shifting since it does not consider consumer flexibility [19]. Therefore, a better pricing approach is required that considers consumer flexibility to maximize demand response [26]. Kaiser et al. [27] presented a framework for designing dynamic prices for reducing power peaks in low-voltage electricity grids as part of a pilot project called OrtsNetz. They used reinforcement learning (RL) to determine the optimal price signals that minimize peak power consumption while considering customer preferences and comfort. A bi-level optimization is solved to obtain a real-time price every 15 min. Although providing valuable insights, the approach lacks the potential for scalability due to its complexity. In addition, the approach focuses on reinforcement learning only and does not include other types of controllers. A more scalable approach for wide implementation of dynamic price is required that can handle different types of controllers.

Based on the literature review, several key research gaps were identified. These include the lack of a method to assess the flexibility potential and price responsiveness of district heating consumers, the limited exploration of dynamic pricing as a tool for activating demand response in district heating, and the need for a scalable approach to designing price signals for demand response purposes. To address these gaps, this paper proposes a novel methodology for designing dynamic price signals that leverage demand flexibility. A stochastic nonlinear flexibility function developed by Junker et al. [12] is used to characterize the energy flexibility of buildings. Unlike existing methods in the literature, which rely on case-specific and complex approaches to determine dynamic prices, this method is more generalizable and computationally efficient, making it easier to scale. Additionally, it can accommodate various types of controllers and is not restricted to a specific consumer type. While the marginal heat production cost method considers only supply availability, this approach also accounts for consumer flexibility, enhancing demand response participation. When fitted, the flexibility function can predict the building's reaction to price, which is then used in an inverse optimization problem to obtain the optimum day-ahead price profile that aims to match the expected demand of consumers with a desired profile. The method is tested on a virtual testbed of a residential neighborhood with 19 buildings. Eventually, critical aspects of this method are discussed. This approach stands out as it is largely scalable, takes into account demand flexibility and consumer behavior, and can handle different types of controllers. Accordingly, dynamic price is also analyzed in this study as a new tariff system for district heating, and its potential for demand response is investigated.

The paper is organized into six sections. In Section 1, an introduction to the topic was given, highlighting the importance of the topic, proposed approach, ongoing studies, and existing literature. In Section 2, the methodology is explained including the equations governing the flexibility function and the optimization problem. In Section 3, the case study used for the analysis and the detailed simulation model (white-box model) are described. Section 4 presents the validation results and simulation results. Key points about this study are discussed in Section 5, mentioning the limitations and future studies. Eventually, Section 6 concludes the paper.

2. Methodology: using flexibility function to design dynamic heat prices

2.1. Energy flexibility model

Buildings can shift the use of a certain amount of energy in time responding to external signal, e.g., price signal, motivation tariff, or carbon intensity of the network [28]. A dynamic price should reflect the actual production costs and, at the same time, motivate consumers to shift their loads accordingly [19]. To design the price signal, system behavior, and energy flexibility of buildings should be first identified. For this, a set of nonlinear stochastic differential equations describing the flexibility function developed by Junker is used [29]. It should be noted that systems must be price-responsive to characterize their flexibility using this method. This means that the buildings should react to the price to some extent (i.e., naturally increase in consumption when the price is low, and vice versa). The model applies concepts of a generalized battery thinking such as a state-of-charge, capacity, etc. to the energy flexible system. The following equation describes the state-of-charge:

$$dX_t = \frac{1}{C}(D_t - B_t - \varepsilon(X_t - \tau))dt + X_t(1 - X_t)\sigma_X dW_t \tag{1}$$

where X_t is the state of charge of the system with a value between 0 and 1, where 0 indicates that the system is in an empty state and has the highest capacity to be charged, and a value of 1 means that the system is fully charged, and no further charging is possible. The parameter C is the amount of flexible energy that can be shifted, D_t shows the expected demand from the system when a specific price is given and B_t corresponds to the baseline demand, which is the demand of the system when there is a fixed price. W_t is Wiener process, σ_X is the intensity of the system noise, and τ is the balancing coefficient. The balancing coefficient denotes the behavior of the system in pushing the state from boundary states to the baseline state. A low balancing coefficient corresponds to a system that is rapidly getting pushed to the baseline state by time, while a high balancing coefficient indicates a system that can stay long in the boundary states. In this application, τ value is found to be at 0.5 based on the realization of system behavior in simulations. In the context of buildings, state of charge of the system (X_t) corresponds to the energy level of the building where a value of 0 means that the building is at the lowest possible state just before experiencing thermal discomfort and a value of 1 corresponds to a state where the building is at the highest energy level, and no more increase in the energy level of the building is possible. Parameter C is the total capacity of flexible energy of the building, which depends on the thermal properties of the building and indoor setpoint limits. A higher setpoint limit range will result in a higher flexible energy (C). Baseline demand (B_t) is the system's demand if the controller is price-ignorant or if a fixed price is given to the system. D_t is the expected demand of the system in the presence of a dynamic price. This is the parameter that the flexibility function would predict, given the baseline demand, predefined price schedule, and trained parameters of the flexibility function. As a default, the state equation considers that when the state of the system deviates due to changes in the price, it tends to get back to the baseline state after some time. However, in the case of heating use in buildings, this assumption does not hold, and it is assumed that when the price is the lowest, consumer keeps the thermostat at a high setpoint no matter for how long, and vice versa. To address this, $\varepsilon(X_t - \tau)$ term is added to the state equation, where τ is assumed to be an average state which is set to 0.5, and ε is a parameter that needs to be tuned to find how fast the building would move towards the baseline over time. Without this term, the prediction of the flexibility function would not be accurate as it is considered in the state equation that the building can only stay at low and high setpoints for a short period of time.

The model assumes that any change from the baseline demand is a function of the state of charge of the system (shown by $f(x)$) or changes in the price applied to the system (shown by $g(x)$). By defining these two functions and normalizing them to be between -1 and $+1$, the following equation would be derived:

$$\delta_t = l(f(X_t;\alpha) + g(u_t;\beta);k) \tag{2}$$

where α and β are function tuning parameters k is called energy flexibility eagerness, indicating the speed of demand changes, u_t is the price signal at time t, and $l(x;k)$ is a scaled logistic function. The logistic function together with k indicate how aggressively the energy flexibility is used. Using the dynamical equation for the state-of-charge (Eq. (1)), the expected demand is calculated as:

$$D_t = B_t + \delta_t \Delta (L(\delta_t > 0)(1 - B_t) + L(\delta_t < 0)B_t) \tag{3}$$

$$L(x > 0) = \begin{cases} 1 & x > 0 \\ 0 & else \end{cases} \tag{4}$$

where Δ is the proportion of flexible demand, which indicates the amount of flexible demand in the baseline demand and L is the indication function, shown in Eq. (4). Eventually, the relationship between the measured demand (Y) and expected demand (D) is as follows:

$$Y_t = D_t + \sigma_Y \varepsilon_t \tag{5}$$

in which Y_t is the measured demand, e.g., the actual energy demand of a building, σ_Y is random error and $\varepsilon_t \sim N(0,1^2)$. Ideally, expected demand (D) should match measured demand (Y). For more information on the equations and the model parameters, readers are referred to Ref. [29]. For simplicity, the relationship between price and demand using flexibility function (FF) can be expressed as follows:

$$D_t = FF(u_t, B_t; \theta) \tag{6}$$

where T is the prediction horizon of the flexibility function, and θ contains all the parameters of the flexibility function. Using observed time series of prices and demand, these parameters can be estimated using methodologies for estimating parameters in discretely and partially observed stochastic differential equations as for instance described in Ref. [30].

2.2. Dynamic price design

Using Eq. (6), the response of a price-responsive system to a price signal can be estimated and evaluated under different circumstances. An optimization algorithm can be used to find the optimum price to minimize the difference between the system response and a desired response, commonly known as an inverse-optimization. The workflow for designing the optimum heat price is illustrated in Fig. 1.

Price profiles (u) generated by the optimization algorithm are evaluated by the trained flexibility function. Flexibility function requires a baseline demand (B) as an input which is the prediction of system behavior without dynamic price. The error between the expected demand (D) and target demand (P_{target}) is used to define reference type controllers for finding the optimum price at any given point in time, which is the cost function to be iteratively minimized. The optimization equations are as follows:

$$\underset{u}{Minimize} \sum_{t=1}^{T} |D(t) - P_{target}(t)| \tag{7}$$

Subject to:

$$u_{min} \leq u(t) \leq u_{max} \quad \forall t \tag{8}$$

$$D = FF(u \mid u \in \{1, 2, \ldots, \mathrm{T}\}, B | B \in \{1, 2, \ldots, \mathrm{T}\}, \theta) \tag{9}$$

where T is the time horizon, and u_{min} and u_{max} are minimum and maximum allowed prices at each time unit.

The aim is to design a dynamic heat price to minimize the difference between the measured demand and the target demand. The target demand is the desired demand profile of the district heating operator, which the operator designs for economic and environmental purposes, such as peak load shaving, which can prevent oversized equipment in

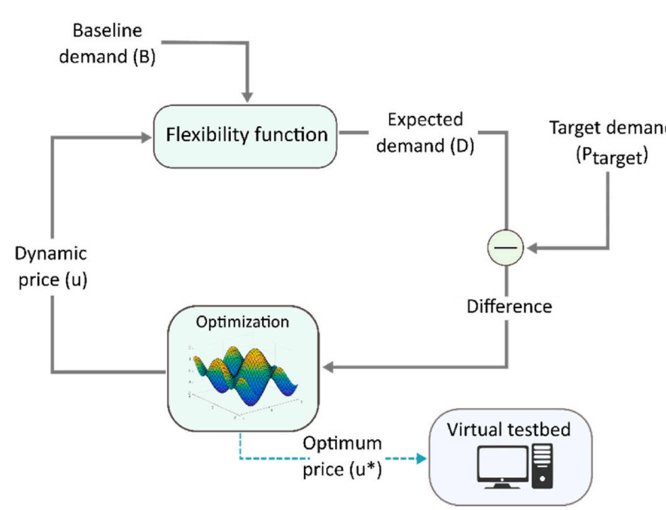

Fig. 1. Workflow for finding optimum price.

the design and improve the integration of heat recovery units and renewable energy sources. Identifying the target demand requires knowledge from the plants and is usually an economic optimization problem, which is out of the scope of this paper. A similar setup for power systems is studied in Ref. [31]. Therefore, a peak load shifting profile is considered as the target demand for the heat price signal design. For the optimization, Genetic Algorithm is used as a heuristic optimization method [32]. The parameters of the algorithm are set after multiple iterations of simulations. Accordingly, maximum iteration is set to 1000, population size is 200, and mutation and crossover percentages are 30 % and 85 %.

To find the optimum heat price, a winter day (i.e. 20th January 2022) is selected for the analysis. The target demand is a peak load shaving profile between 5:00–9:00 and 16:00–20:00. A short pre-charging period is considered before the peak periods, and the rest is the same as the baseline demand. Finally, to assess the effect of using dynamic heat price and demand response, two cases are identified: price-responsive (reacting to the dynamic heat price) and price-ignorant (baseline).

2.3. Peak load reduction

To quantify the peak load reduction potential of using dynamic price, the following equation is used:

$$Peak\ load\ reduction\ (\%) = \frac{Y_{DR} - B_{DR}}{B_{DR}} \times 100 \tag{10}$$

where Y_{DR} and B_{DR} are the measured demand and baseline demand during a peak load reduction period.

2.4. Model accuracy

Before using the flexibility function, it should be tested and validated using measurement data. The KPIs used for validation are Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) methods, commonly used for evaluating forecast and model accuracy. These metrics are calculated as:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - D_i)^2}{n}} \tag{11}$$

$$\mathrm{MAPE} = \frac{100}{n} \sum_{i=1}^{n} \frac{Y_i - D_i}{Y_i} \tag{12}$$

where n is the number of data points and Y_i and D_i are the measurement data and model output, respectively.

3. Case study

The above methodology is applied to a case study to demonstrate the design process of dynamic heat prices and their impact. The case study is a neighborhood consisting of 19 multi-family residential blocks with 432 apartments in total, located in Sønderborg, Denmark. The buildings were built around 1970 and renovated multiple times. The buildings have similar floor plans and differ only in orientation, floor area, and number of floors (2 or 3). The buildings are part of the ARV project's Danish demonstration site [33]. An aerial view of the case study and a photo of one of the buildings are shown in Fig. 2. There are 9 substations distributing heat to 19 building blocks. Each substation is labeled with its corresponding block in the same color, e.g., the substation K9 provides heat to the four buildings marked in yellow.

3.1. Heating system

Sønderborg Varme is the local district heating company providing heat to the case study neighborhood [34]. On the space heating side, district heating supply water is mixed by the return water of the radiators (by controlling the mixing ratio) to provide the proper forward temperature for the radiators. A Weather Compensation Curve (WCC) determines the appropriate forward temperature. The radiators are equipped with Thermostatic Radiator Valves (TRV), automatically adjustable valves that maintain indoor temperature at a specific range. The heating controllers are controlling the indoor temperature by changing TRV setpoints. Indoor temperature is assumed to change between 18 °C and 26 °C according to the price. In this study, the indoor setpoint is determined by the dynamic heat price using a fuzzy system. By using the fuzzy controller, a smoother relationship between the price and setpoint temperature can be established, making it easier for the flexibility function to find the related parameters. The utilized fuzzy system is a simple Mamdani function with price as the input and indoor setpoint temperature as the output. Five trapezoid and Gaussian membership functions are determined for the input and the output based on user knowledge. Fig. 3 shows the relationship between heat price, indoor setpoints, and forward temperature settings.

3.2. Data collection

Data from multiple sources were collected to complete the model development.

- Substation heat consumption

Daily substation heat consumption measurements are collected by heat meters in the substations, which consist of space heating, domestic hot water consumption, and circulation losses. These measurements are used for model validation.

Fig. 2. Left - Arial view of the neighborhood and district heating substations in Sønderborg. Colors represent individual substations. Right - Photo taken outside one of the buildings.

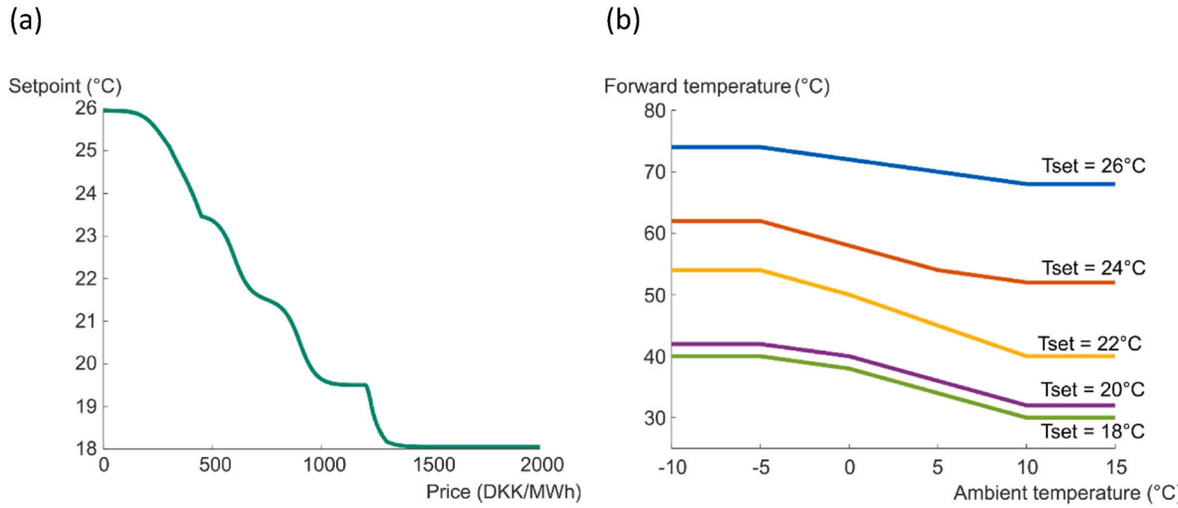

Fig. 3. (a) Determined indoor setpoint based on heating price, (b) SH supply temperature as a function of indoor setpoint and ambient temperature.

- Weather data

The weather data consists of hourly ambient temperature values, relative humidity, wind speed and direction, and solar irradiation, measured in the Sønderborg airport weather station. The data shows that Sønderborg is a heating-dominated city with ambient temperatures between -4 °C and 29.3 °C. Solar irradiation intensity reaches 889.4 W/m^2 in June but is very low in the winter season. Wind speed has high variations, rising up to 17 m/s. The data is used both in the validation phase and simulation phase.

- Building properties

Since the exact information on the building construction elements was unavailable, valid datasets were used, including the Danish Building and Housing Register (BBR) [35], the Danish Building Standard (DS/EN 15251) [36] and the TABULA project [37] to create building models. The building components used for modelling are listed in Table 1. The air infiltration rate is set as 4 $l/s.m^2$ according to Ref. [38].

3.3. Characterizing flexibility function for the neighborhood

As the implementation cannot be conducted physically in the case study, we have built a virtual platform (a white-box model) as the digital twin of the neighborhood.

3.3.1. White-box model as a virtual testbed

Modelica is used to create the neighborhood model using the Dymola interface [39]. The model is composed of a DH plant, substations, and buildings. At the plant level, hot water with the desired temperature and pressure is produced and dispatched to the network. The high-pressure hot water then enters each substation for heating purposes. Here, the water temperature and pressure are reduced by heat exchangers and mixing shunts. After this, hot water with desired conditions is forwarded to radiators and domestic hot water tanks. An illustration of the district model is shown in Fig. 4.

The "MixedAir" component from Modelica Buildings Library is used to create the building thermal models. To model the dynamic behavior of Thermostatic Radiator Valves (TRV), a modified version of the "TwoWayTRV" component from IDEAS library is used [40]. This component models the position of the valve using a smooth Heaviside function.

Table 1
Components used in creating building models.

Component	Materials (thickness)
Roof	Roof tiles (59 mm) Insulation (300 mm) Hollow core concrete (270 mm)
Exterior wall	Brick (108 mm) Insulation (375 mm) Aerated concrete (100 mm)
Floor/ceiling	Concrete (220 mm) Insulation (93 mm) Concrete (80 mm) Oak planks (14 mm)
Ground floor	Insulation (350 mm) Concrete (120 mm)
Windows	Clear double glazing with air
Internal wall	Concrete (200 mm)

- Assumptions and simplifications

Since white-box models intend to represent the real case with high accuracy, they usually require detailed information. In this context, parameters such as radiator sizes, furniture thermal mass, occupancy presence, clogs in heating systems, window curtain positions are challenging to collect, especially for a district-level study. Therefore, some simplifications need to be made.

Thermal zones: Each floor is assumed to be a single thermal zone with an equivalent hydronic radiator for the whole floor. Therefore, a 3-floor building would be modeled by connecting three thermal zone models in Dymola.

Heat loss: In the case study neighborhood, the distance between the blocks is relatively small, so the heat loss to the soil would be negligible. However, heat loss from the substation in the basement increases the basement's temperature. In the model, this heat loss is modeled as a constant heat source in the basement.

Thermal delay: District heating networks are usually larger than just a small neighborhood and cover larger areas. Since this study is limited to analyzing only a small part of the whole district heating network, thermal delay within the pipes was neglected due to small distances.

DH supply temperature: As the focus of the study is on the consumer side the district heating supply temperature is assumed to be fixed.

3.3.2. Virtual experiment workflow

We implement the method for designing dynamic prices (Section 2.2) on the white-box model as the virtual experimental platform, so the measured demand (Y) would be the model output. To characterize the flexibility function, datasets of price (u), baseline demand (B) and measured demand (Y) over a long enough period (i.e. a period that

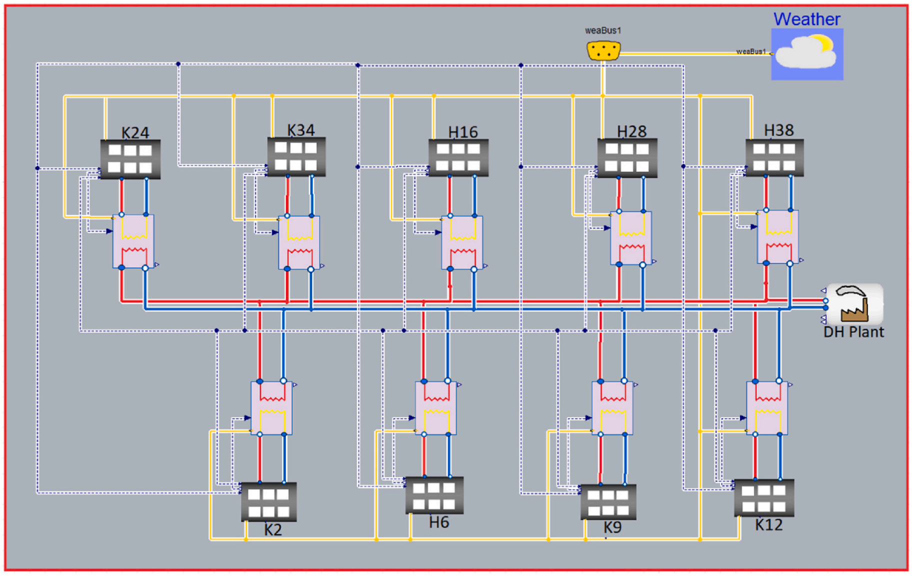

Fig. 4. White-box model of the neighborhood created using Dymola.

includes different varieties of input values) are required. The workflow for building such a dataset is shown in Fig. 5.

The white-box model of the neighborhood is first exported as a Functional-MockUp-Unit (FMU) and then sent to Simulink to integrate with control systems. The Fuzzy controller is modeled in Simulink and connected to the FMU. The controller receives heat price (u) and responds accordingly and sends the response (i.e. radiator control signal) to the FMU block, which then gives the output, the measured demand (Y), which is the reaction of the neighborhood to the price. In reality, measured demand (Y) is the measured demand of neighborhood, but here, since there is no physical implementation available, the white-box model output represents measurement demands. To calculate the baseline, the controller block is removed, and the output of the FMU becomes the baseline heat consumption in the absence of price-responsive controllers.

4. Results

4.1. Validation of white-box model

For a well-tuned model, the error between the simulation results and real measurements should be within an acceptable range. To assess the model accuracy, model validation is performed in January 2022 and the results are shown in Fig. 6.

The validation was performed for six substations out of nine since the collected data was available only for six substations during this period. The simulation is performed using Dymola, with a timestep of 15 min and an error tolerance of 0.001 for convergence between intervals. Results show a Mean Absolute Percentage Error (MAPE) value of 4.8 %–8.9 % for the substations.

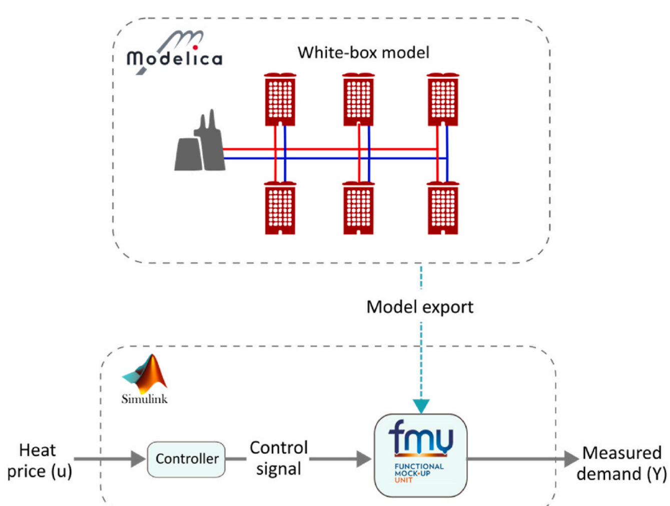

Fig. 5. Workflow for model exchange and controller implementation.

4.2. Flexibility function of the case study neighborhood

The accuracy of the flexibility function model is examined using measured demand. To fit the flexibility function to the neighborhood, a training dataset is required, as mentioned in 3.4. The dataset should contain baseline demand (B), dynamic heat price (u) and measured demand (Y) for a period, as shown in Fig. 5. Estimating the parameters of the flexibility function (i.e. θ in Eq. (6)) requires solving an optimization problem to retrieve a model that can provide accurate predictions of system response to price at different conditions. A training dataset containing a baseline demand, price signal, and the response of the energy flexible system is required to obtain flexibility function parameters. Accordingly, the optimization problem is solved to find the parameters that can best describe the energy flexibility model and system behavior. More information regarding the optimization process is given in Ref. [12]. An iterative approach is used to fit the model to the data and find the flexibility function parameters. Accordingly, heat meter measurements of February 2022 are used as the baseline demand (B) in the training dataset. Since the heat price is currently flat in Denmark, the dynamic electricity price from the Nordpool Day-ahead market for DK1 was taken as the dynamic heat price (u) [41]. The model takes the dynamic price and calculates the measured demand accordingly (Y). The dataset is shown in Fig. 7.

Indoor temperatures are found to follow the setpoint accurately. When the setpoint drops to the lowest value, e.g., the highlighted area in the figure between days 21 and 23, indoor temperature requires more time to reach the setpoint due to the thermal intertia of the buildings. This indicates that the heating system can be turned off in high-price periods with the indoor temperature staying within the temperature comfort zone. In addition, the heat demand profile shows a good dependency on the heat price. For example, drops in the prices on days 18 and 20 are followed by spikes in demand in the same periods. To

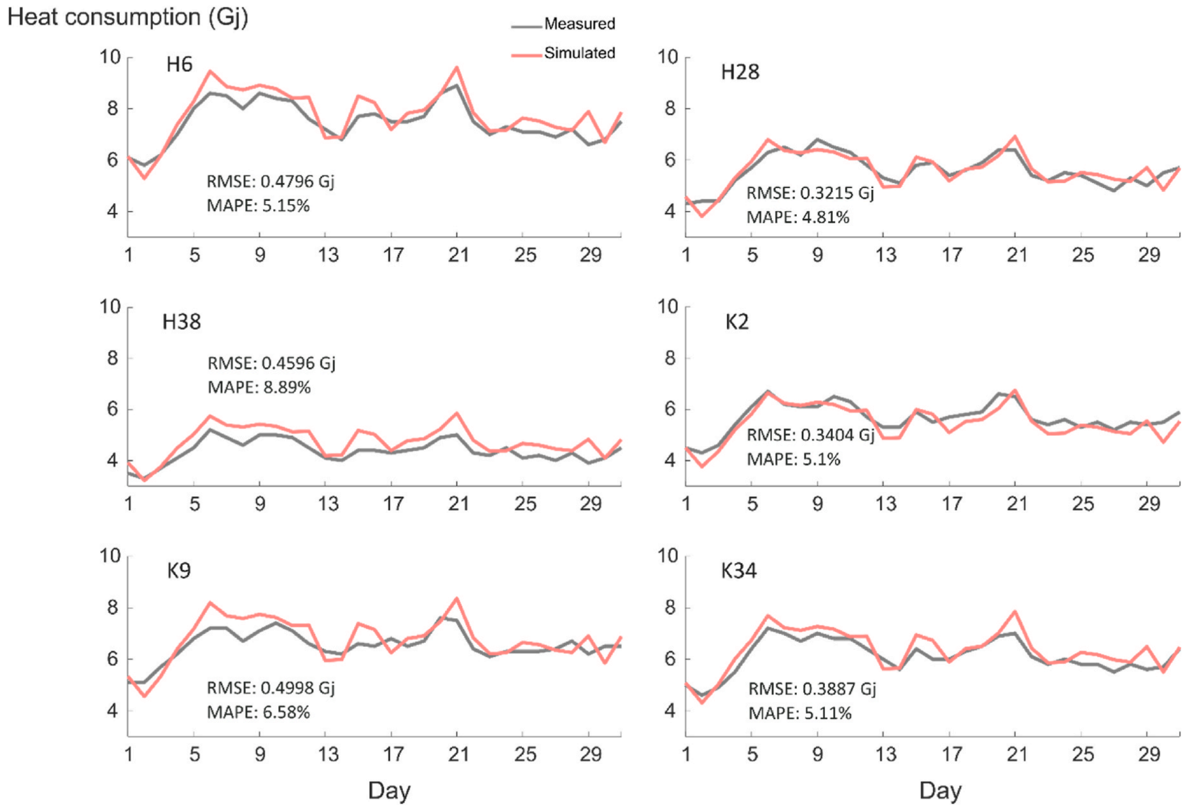

Fig. 6. Model validation results for six substations, conducted in January 2022.

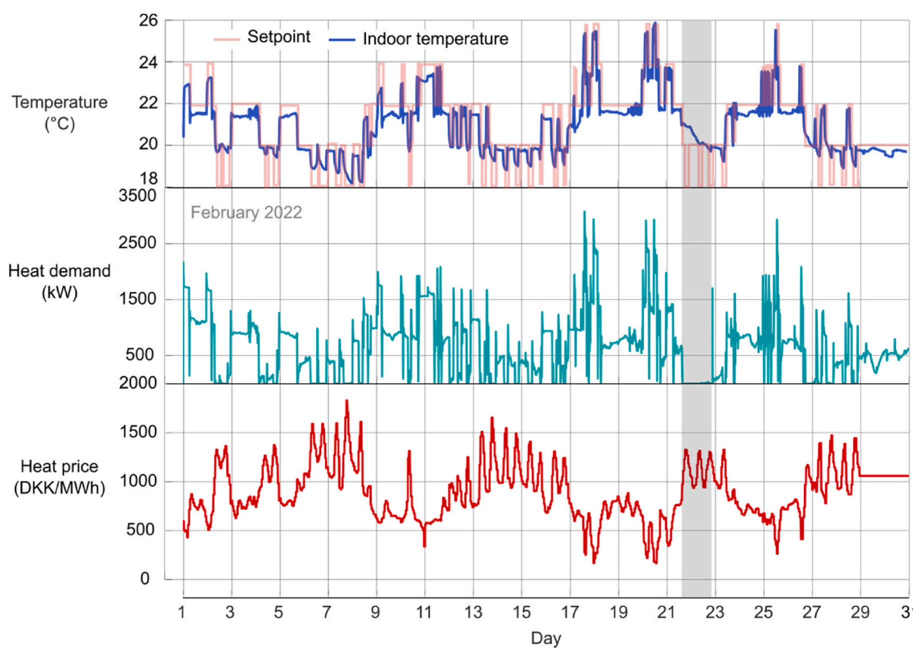

Fig. 7. Dataset used for training the flexibility function. **Top** indoor temperature and setpoints of one of the buildings, **middle** Demand of the neighborhood, and **bottom** dynamic price, taken from Nordpool DK1 day-ahead electricity price.

calculate the baseline demand (B), the same fixed TRV settings of 21 °C are used instead of the fuzzy controllers, and simulation is repeated in the same period. Eventually, this dataset is used to train the flexibility function. Fig. 8 compares the flexibility function model output (D) together with the result of the white-box model (Y) given the random heat price (u), for one week.

The prediction demand by flexibility function (D) closely matches the measured demand (Y) with an MAPE of 6.7 % and RMSE equal to 249.8 kW. The fitted flexibility function parameters are given in Table 2. For information about the parameters can be found in Ref. [14].

Δ value of 1 means that the function assumes all the demand in this neighborhood is flexible. α and β are parameters of the f and g functions,

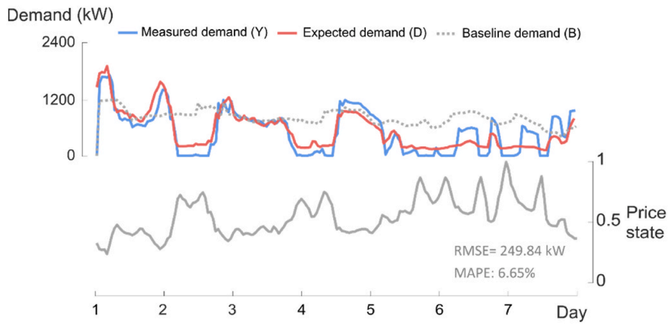

Fig. 8. Expected demand (D) profile of the neighborhood, compared with measured demand (Y), applying variable price (u) in one week.

Table 2
Parameter estimates of the flexibility function model for the neighborhood.

Parameters	values	Description
C	20	Total amount of energy available for flexible use
Δ	1	Proportion of flexibility demand
k	4.697485	Energy flexibility eagerness
ε	0.674467	State deviation speed
α_1	0.5	Function f parameters
α_2	0.016792	
α_3	0.756731	
α_4	0.226477	
β_1	0.319242	Function g parameters
β_2	0.122078	
β_3	0.070485	
β_4	0.108339	
β_5	0.160450	
β_6	0.219406	
β_7	0	

which constitutes essential parts of the flexibility function as described in Eq (2). Readers are recommended to refer to Ref. [14] for further details on the flexibility function parameters.

4.3. Optimum heat price

In this section, the results of using the trained flexibility function for designing the optimum price for the neighborhood, using the method shown in Section 2.2 are presented.

Fig. 9 shows that the price is high in the peak periods and low in the pre-charging hours. The magnitudes of the dynamic heat price depend on the flexibility of buildings and the target demand value. If buildings are not flexible enough, the resulting dynamic heat price is expected to have more extreme prices. The fluctuations in measured demand (Y), for example at 15:00 is due to controllers in the model reacting to sudden

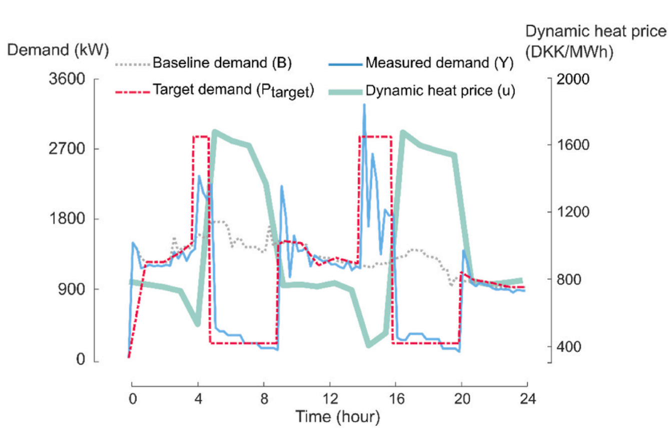

Fig. 9. The optimum heat price for the neighborhood.

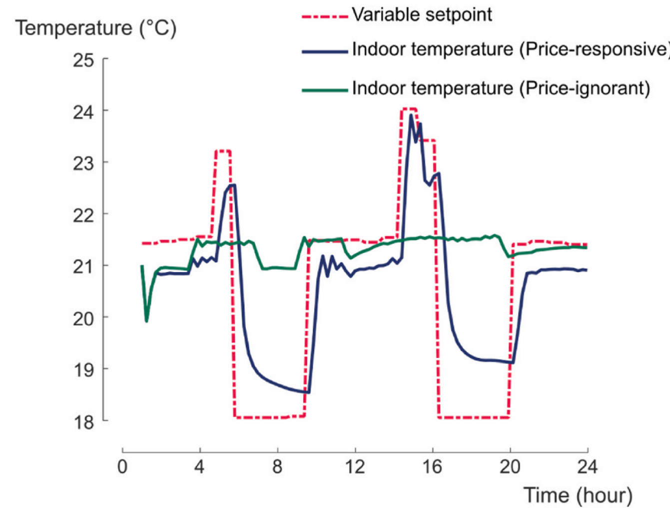

Fig. 10. Average indoor temperature of the buildings for both price-responsive and price-ignorant cases.

changes in thermostat settings. Using the dynamic heat price, neighborhood demand is matched with the target demand with an MAPE value of 16.6 %. The measured demand (Y) profile shows a notable peak reduction during morning peaks (5:00–9:00) and evening peaks (16:00–20:00), due to increased prices. Accordingly, the average peak load reduction values during morning and evening peak periods are 84.4 % and 81.7 %, respectively. Table 3 shows the total heating consumption and the heating costs for the two cases, and Fig. 10 shows the indoor temperatures and the setpoint.

The price-responsive system provided 46.6 % savings in the heating cost for the neighborhood. Still, the thermal comfort state would be satisfied since the controllers are bound to be between the thermal comfort limits (18 °C–26 °C according to the Danish building standard [42]).

The figure below shows the average indoor temperature of all buildings in the neighborhood for both price-responsive and price-ignorant cases.

The red dashed line shows the variable setpoint determined by the fuzzy controller, given the optimum dynamic price. The indoor temperature in the price-responsive system follows the setpoint by automatically adjusting TRVs. The slight bias between the setpoint and indoor temperature is due to the 0.5 °C dead band of the TRVs, which means that the TRVs keep the indoor temperature within 0.5 °C of the setpoint. The indoor temperature does not reach 18 °C mainly due to the high thermal inertia of the buildings. The indoor temperature for the price-ignorant case is relatively constant throughout the period due to a fixed setpoint of 21 °C. The average indoor temperatures for the price-ignorant and price-responsive cases are 21.3 °C and 20.6 °C, respectively.

5. Discussion

In this study, the methodology of designing dynamic heat price is tested on a virtual testbed of a neighborhood. Implementing and testing dynamic heat price in real-world district heating systems is challenging due to the complexity of factors involved and uncontrolled study

Table 3
Heating consumption and the total heating cost for the price-ignorant and price-responsive cases.

	Heating consumption (MWh)	Heating cost (DKK)
Price-ignorant	27.6	27,667
Price-responsive	20.4	14,772

environment. For example, in practice, heat consumers might react uniquely to dynamic heat prices, making predictions challenging. Additionally, the current infrastructure is not equipped for broadcasting the prices, control, monitoring, etc. Therefore, we conducted the study on a representative, detailed model (white-box model) of the neighborhood. This study aimed to reveal the potential of buildings in a neighborhood for demand response in ideal situations. Therefore, the indoor temperature range was set to 18 °C–26 °C. In reality, the accepted temperature bound might be much smaller. Additionally, the controller type chosen here perfectly complies with dynamic prices. Most buildings do not have smart thermostats and will not accept such direct controls. Although these assumptions deviate from reality, they provided valuable insights into the maximum potential of buildings for demand response and the benefits of dynamic price.

The role of climate in the proposed method is to determine the baseline demand value. The highest influencing parameter is the controller type, which can be a rule-based control, manual control of occupants, or automatic controller type. In reality, full automation may not be feasible, and end-users might have a notable influence on reactions. Although this would add uncertainty and stochasticity to the data, the stochastic flexibility function can still fit a reasonable model if enough data is given. Still, end-user willingness to participate in demand response events can change from time to time and can create errors in determining the optimum price. An adaptive version of the flexibility function can be a solution for this, where the parameters of the flexibility function update accordingly [43]. A follow-up study is required to consider different consumer types with different controllers better to understand the role of occupant behavior in this. This study considered a specific case study with a mild climate to test the proposed methodology. However, this does not reduce the generalizability of the method. The method can be applied to any energy-flexible system with any sort of energy demand, including cooling and electricity. In general, to apply the method to any energy-flexible system, the following general steps should be taken.

1) **Baseline model:** A model for estimating baseline demand should be established, which can be a dynamic model, a statistical model based on historical data, etc.
2) **System response to price:** The actual response of the system to dynamic price in different conditions should be measured. This information, together with the baseline demand, would be used to fit the flexibility function and estimate the parameters.
3) **Target demand:** A target demand should be available to design dynamic prices accordingly, which can be a load-shifting profile, peak-shaving profile, or maximizing self-consumption.

Domestic Hot Water (DHW), the main contributor to district heating peaks, is not included in this study. However, experiences in practice show that the flexibility potential of DHW is limited due to the limited size of the DHW tank at the substation; thus, it is much smaller than the flexibility the heating system can offer [44].

Target demand is essential as the price is calculated based on that. District heating operators determine the target demand, which can be derived from internal optimization of the plant productions. It can be prepared to shape the network's total demand for multiple purposes, e.g., preventing peaks, reducing return water temperature, promoting the economic benefit of plants, and maximizing self-consumption. Focusing on how to find the target demand was out of the scope of this paper, but it is an essential factor for district heating operators. Basically, if the aim is to minimize production costs, the target demand profile would look like the reverse of the marginal heat production cost profile. But, if the aim is peak load shifting, environmental benefits, or solving network congestion, the target demand would be different than the marginal heat production cost.

The dynamic heat price used in this study is based on obtaining a fixed day-ahead price profile and sending it to the users. This makes the approach sensitive to the baseline prediction when calculating the price profile. Another approach is to apply an updating scheme as real-time pricing, where the optimum price is first determined, and real-time updates are conducted based on measured reactions of users and weather conditions. Future studies can compare the methods and focus on the uncertainty of baseline demand predictions.

6. Conclusion

This study proposed a methodology for designing dynamic heat prices for demand response purposes, considering demand flexibility and consumer reaction. The approach relies on using a stochastic nonlinear flexibility function to characterize the energy flexibility of buildings in response to the dynamic price. An inverse optimization problem is used to determine the optimal day-ahead price profile that balances consumer demand with the district heating operator's target demand. The methodology was applied to the case study of a neighborhood of 19 residential apartment blocks in Sønderborg, Denmark. Results for the neighborhood show that applying the designed dynamic heat price could match demand and supply with a MAPE of 16.6 %, and the morning and evening peak loads were reduced by up to 84.4 %. In addition, consumer heat costs were reduced by 46.6 % by changing thermostat settings using a fuzzy controller.

This paper builds a good foundation for district heating operators and consumers to understand the potential benefits of shifting from the current flat price mechanism to a dynamic price. More studies, however, are required to analyze a neighborhood of mixed controllers and behavior ones, better reflecting the real-world situation to understand the expected outcome. In general, this study shows that if designed properly, dynamic price can be an effective strategy for the wide implementation of demand response in district heating.

CRediT authorship contribution statement

Reza Mokhtari: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Data curation, Conceptualization. **Rune Grønborg Junker:** Supervision, Software, Methodology, Formal analysis, Conceptualization. **Henrik Madsen:** Writing – review & editing, Supervision, Project administration, Funding acquisition, Formal analysis, Conceptualization. **Rongling Li:** Writing – review & editing, Supervision, Resources, Project administration, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The authors do not have permission to share data.

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